4.9 - Solving Systems of Linear DEs by Elimination

The DE $y' + 2y + 3x = -1$ can be written in terms of the differential operator D, where D represents a derivative. In this form, the DE becomes $Dy + 2y + 3x = -1$. This can also be written $(D+2)y + 3x = -1$. Our work in this lesson makes use of this notation. For this lesson, x and y are functions of t, so $Dx = \dfrac{dx}{dt}$, $Dy = \dfrac{dy}{dt}$, and $D^2x = \dfrac{d^2x}{dt^2}$, etc.

Solve the given system of differential equations by elimination.

Example:

$$\begin{aligned} (D+1)x \ &+(D-1)y \ = 2 \\ 3x \ &+(D+2)y \ = -1 \end{aligned}$$

Example:

$$\begin{aligned} \frac{dx}{dt} + \frac{dy}{dt} &= e^t \\ -\frac{d^2x}{dt^2} + \frac{dx}{dt} + x + y &= 0 \end{aligned}$$

Example:

$$\begin{aligned} Dx \qquad\qquad\quad +z &= e^t \\ (D-1)x \quad +Dy \quad +Dz &= 0 \\ x \quad +2y \quad +Dz &= e^t \end{aligned}$$

Example:

$$\frac{dx}{dt} = -x + z$$
$$\frac{dy}{dt} = -y + z$$
$$\frac{dz}{dt} = -x + y$$